‡012List of functionssection*.31 §

ordpy

Release 1.0

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CONTENTS

ordpy is a pure Python module[1] that implements data analysis methods based on Bandt and Pompe's[2] symbolic encoding scheme.

Note: If you have used ordpy in a scientific publication, we would appreciate citations to the following reference[1]:

- A. A. B. Pessa, H. V. Ribeiro, ordpy: A Python package for data analysis with permutation entropy and ordinal network methods, Chaos 31, 063110 (2021). arXiv:2102.06786

```
@misc{pessa2021ordpy,
  title         = {ordpy: A Python package for data analysis with permutation␣
↪entropy and ordinal network methods},
  author        = {Arthur A. B. Pessa and Haroldo V. Ribeiro},
  journal       = {Chaos: An Interdisciplinary Journal of Nonlinear Science},
  volume        = {31},
  number        = {6},
  pages         = {063110},
  year          = {2021},
  doi           = {10.1063/5.0049901},
 }
```

ordpy implements the following data analysis methods:

Released on version 1.0.0 (February 2021):

- Permutation entropy for time series[2] and images[3];
- Complexity-entropy plane for time series[4],[5] and images[3];
- Multiscale complexity-entropy plane for time series[6] and images[7];
- Tsallis[8] and Rényi[9] generalized complexity-entropy curves for time series and images;
- Ordinal networks for time series[10],[11] and images[12];
- Global node entropy of ordinal networks for time series[13],[11] and images[12].
- Missing ordinal patterns[14] and missing transitions between ordinal patterns[11] for time series and images.

[1] Pessa, A. A., & Ribeiro, H. V. (2021). ordpy: A Python package for data analysis with permutation entropy and ordinal networks methods. Chaos, 31, 063110.

[2] Bandt, C., & Pompe, B. (2002). Permutation entropy: A Natural Complexity Measure for Time Series. Physical Review Letters, 88, 174102.

[3] Ribeiro, H. V., Zunino, L., Lenzi, E. K., Santoro, P. A., & Mendes, R. S. (2012). Complexity-Entropy Causality Plane as a Complexity Measure for Two-Dimensional Patterns. PLOS ONE, 7, e40689.

[4] Lopez-Ruiz, R., Mancini, H. L., & Calbet, X. (1995). A Statistical Measure of Complexity. Physics Letters A, 209, 321-326.

[5] Rosso, O. A., Larrondo, H. A., Martin, M. T., Plastino, A., & Fuentes, M. A. (2007). Distinguishing Noise from Chaos. Physical Review Letters, 99, 154102.

[6] Zunino, L., Soriano, M. C., & Rosso, O. A. (2012). Distinguishing Chaotic and Stochastic Dynamics from Time Series by Using a Multiscale Symbolic Approach. Physical Review E, 86, 046210.

[7] Zunino, L., & Ribeiro, H. V. (2016). Discriminating Image Textures with the Multiscale Two-Dimensional Complexity-Entropy Causality Plane. Chaos, Solitons & Fractals, 91, 679-688.

[8] Ribeiro, H. V., Jauregui, M., Zunino, L., & Lenzi, E. K. (2017). Characterizing Time Series Via Complexity-Entropy Curves. Physical Review E, 95, 062106.

[9] Jauregui, M., Zunino, L., Lenzi, E. K., Mendes, R. S., & Ribeiro, H. V. (2018). Characterization of Time Series via Rényi Complexity-Entropy Curves. Physica A, 498, 74-85.

[10] Small, M. (2013). Complex Networks From Time Series: Capturing Dynamics. In 2013 IEEE International Symposium on Circuits and Systems (ISCAS2013) (pp. 2509-2512). IEEE.

[11] Pessa, A. A., & Ribeiro, H. V. (2019). Characterizing Stochastic Time Series With Ordinal Networks. Physical Review E, 100, 042304.

[12] Pessa, A. A., & Ribeiro, H. V. (2020). Mapping Images Into Ordinal Networks. Physical Review E, 102, 052312.

[13] McCullough, M., Small, M., Iu, H. H. C., & Stemler, T. (2017). Multiscale Ordinal Network Analysis of Human Cardiac Dynamics. Philosophical Transactions of the Royal Society A, 375, 20160292.

[14] Amigó, J. M., Zambrano, S., & Sanjuán, M. A. F. (2007). True and False Forbidden Patterns in Deterministic and Random Dynamics. Europhysics Letters, 79, 50001.

Released on version 1.1.0 (January 2023):

- Weighted permutation entropy for time series[15] and images;
- Fisher-Shannon plane for time series[16] and images;
- Permutation Jensen-Shannon distance for time series[17] and images;
- Four pattern permutation contrasts (up-down balance, persistence, rotational-asymmetry, and up-down scaling.) for time series[18];
- Smoothness-structure plane for images[19].

References

[15] Fadlallah B., Chen, B., Keil A. & Príncipe, J. (2013). Weighted-permutation entropy: a complexity measure for time series incorporating amplitude information. Physical Review E, 97, 022911.

[16] Olivares, F., Plastino, A., & Rosso, O. A. (2012). Contrasting chaos with noise via local versus global information quantifiers. Physics Letters A, 376, 1577–1583.

[17] Zunino L., Olivares, F., Ribeiro H. V. & Rosso, O. A. (2022). Permutation Jensen-Shannon distance: A versatile and fast symbolic tool for complex time-series analysis. Physical Review E, 105, 045310.

[18] Bandt, C. (2023). Statistics and contrasts of order patterns in univariate time series, Chaos, 33, 033124.

[19] Bandt, C., & Wittfeld, K. (2023). Two new parameters for the ordinal analysis of images, Chaos, 33, 043124.

CHAPTER

ONE

INSTALLING

Ordpy can be installed via the command line using

```
pip install ordpy
```

or you can directly clone its git repository:

```
git clone https://github.com/arthurpessa/ordpy.git
cd ordpy
pip install -e .
```

CHAPTER

TWO

BASIC USAGE

We provide a notebook illustrating how to use ordpy. This notebook reproduces all figures of our article[1]. The code below shows simple applications of ordpy.

```
#Complexity-entropy plane for logistic map and Gaussian noise.

import numpy as np
import ordpy
from matplotlib import pylab as plt

def logistic(a=4, n=100000, x0=0.4):
    x = np.zeros(n)
    x[0] = x0
    for i in range(n-1):
        x[i+1] = a*x[i]*(1-x[i])
    return(x)

time_series = [logistic(a) for a in [3.05, 3.55, 4]]
time_series += [np.random.normal(size=100000)]

HC = [ordpy.complexity_entropy(series, dx=4) for series in time_series]


f, ax = plt.subplots(figsize=(8.19, 6.3))

for HC_, label_ in zip(HC, ['Period-2 (a=3.05)',
                            'Period-8 (a=3.55)',
                            'Chaotic (a=4)',
                            'Gaussian noise']):
    ax.scatter(*HC_, label=label_, s=100)

ax.set_xlabel('Permutation entropy, $H$')
ax.set_ylabel('Statistical complexity, $C$')

ax.legend()
```

```
#Ordinal networks for logistic map and Gaussian noise.

import numpy as np
import igraph
import ordpy
```

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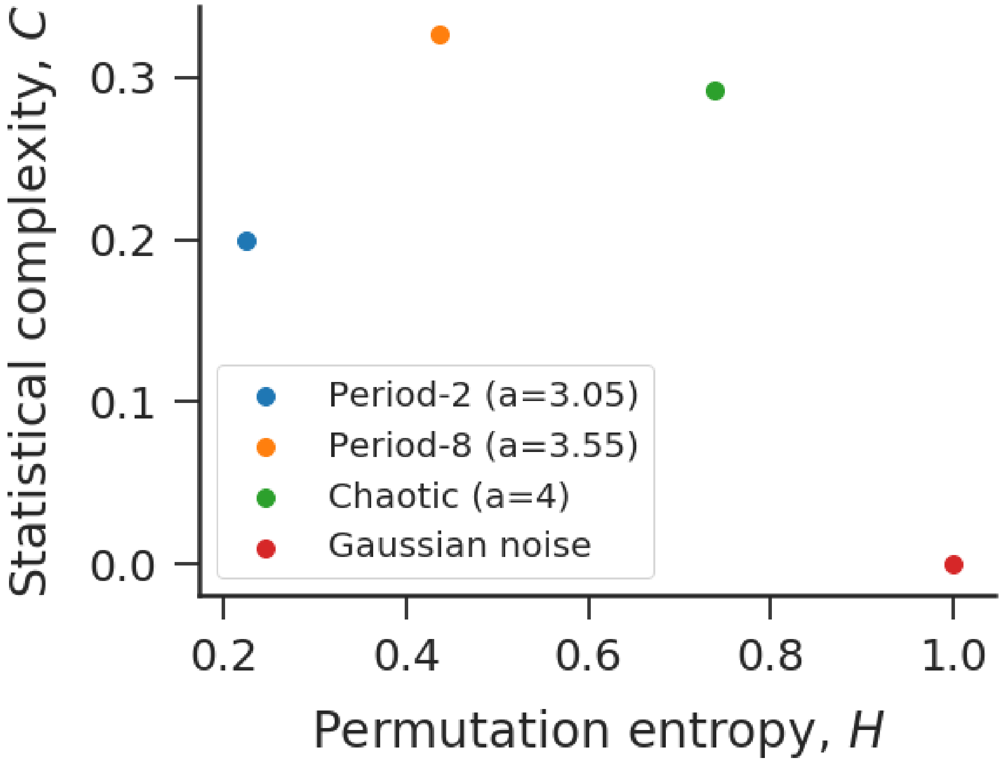

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```
from matplotlib import pylab as plt
from IPython.core.display import display, SVG

def logistic(a=4, n=100000, x0=0.4):
    x = np.zeros(n)
    x[0] = x0
    for i in range(n-1):
        x[i+1] = a*x[i]*(1-x[i])
    return(x)

time_series = [logistic(a=4), np.random.normal(size=100000)]

vertex_list, edge_list, edge_weight_list = list(), list(), list()
for series in time_series:
    v_, e_, w_   = ordpy.ordinal_network(series, dx=4)
    vertex_list += [v_]
    edge_list   += [e_]
    edge_weight_list += [w_]

def create_ig_graph(vertex_list, edge_list, edge_weight):

    G = igraph.Graph(directed=True)

    for v_ in vertex_list:
        G.add_vertex(v_)
```

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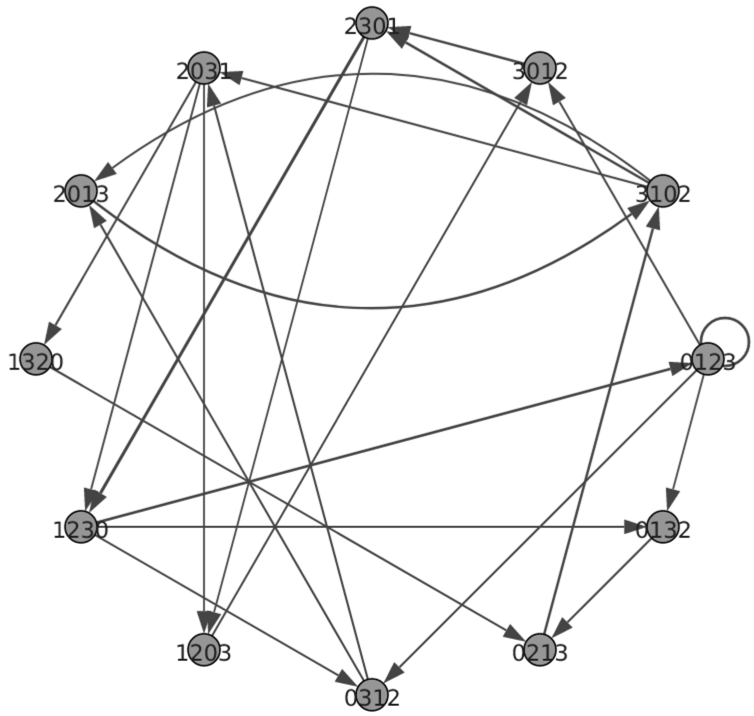
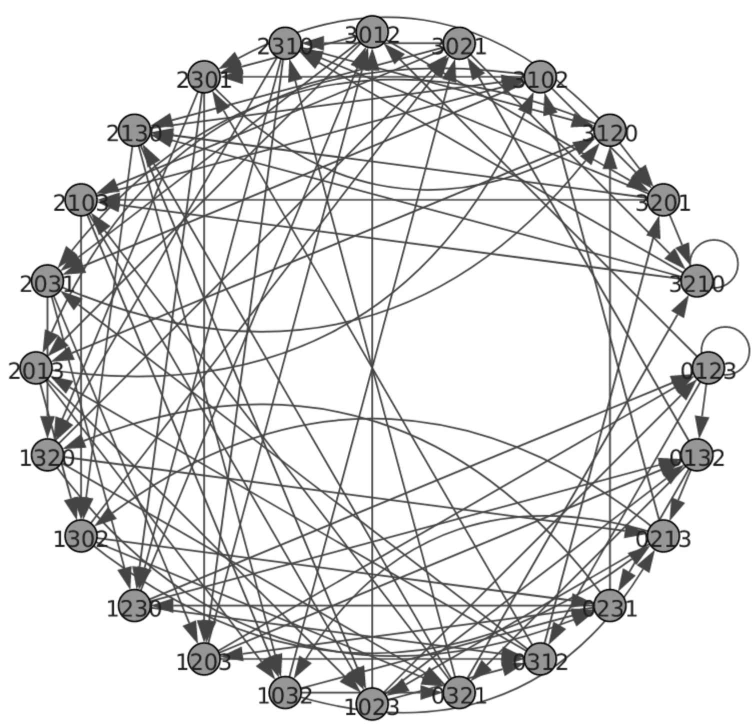
```
    for [in_, out_], weight_ in zip(edge_list, edge_weight):
        G.add_edge(in_, out_, weight=weight_)

    return G

graphs = []

for v_, e_, w_ in zip(vertex_list, edge_list, edge_weight_list):
    graphs += [create_ig_graph(v_, e_, w_)]

def igplot(g):
    f = igraph.plot(g,
                    layout=g.layout_circle(),
                    bbox=(500,500),
                    margin=(40, 40, 40, 40),
                    vertex_label = [s.replace('|','') for s in g.vs['name']],
                    vertex_label_color='#202020',
                    vertex_color='#969696',
                    vertex_size=20,
                    vertex_font_size=6,
                    edge_width=(1 + 8*np.asarray(g.es['weight'])).tolist(),
                   )
    return f

for graph_, label_ in zip(graphs, ['Chaotic (a=4)',
                                   'Gaussian noise']):
    print(label_)
    display(SVG(igplot(graph_)._repr_svg_()))
```

```
Chaotic (a=4)
```




```
Gaussian noise
```



CHAPTER

THREE

LIST OF FUNCTIONS

ordpy.**complexity_entropy**(*data*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *probs=False*, *tie_precision=None*)

Calculates permutation entropy[Page 1, 2] and statistical complexity[Page 1, 4,Page 1, 5] using an ordinal distribution obtained from data.

Parameters

- **data** (*array*) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ or an ordinal probability distribution (such as the ones returned by *ordpy.ordinal_distribution()*).
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **probs** (*boolean*) – If *True*, it assumes **data** is an ordinal probability distribution. If *False*, **data** is expected to be a one- or two-dimensional array (default: *False*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns

Values of normalized permutation entropy and statistical complexity.

Return type

tuple

Examples

```
>>> complexity_entropy([4,7,9,10,6,11,3], dx=2)
(0.9182958340544894, 0.06112816548804511)
>>>
>>> p = ordinal_distribution([4,7,9,10,6,11,3], dx=2, return_missing=True)[1]
>>> complexity_entropy(p, dx=2, probs=True)
(0.9182958340544894, 0.06112816548804511)
>>>
>>> complexity_entropy([[1,2,1],[8,3,4],[6,7,5]], dx=2, dy=2)
(0.3271564379782973, 0.2701200547320647)
>>>
>>> complexity_entropy([1/3, 1/15, 4/15, 2/15, 1/5, 0], dx=3, probs=True)
(0.8314454838586238, 0.16576716623440763)
```

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```
>>>
>>> complexity_entropy([[1,2,1,4],[8,3,4,5],[6,7,5,6]],dx=3, dy=2)
(0.21070701155008006, 0.20704765093242872)
```

ordpy.**fisher_shannon**(*data*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *probs=False*, *tie_precision=None*)

Calculates permutation entropy[Page 1, 2] and Fisher information[Page 2, 16] using an ordinal distribution obtained from data.

Parameters

- **data** (*array*) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ or an ordinal probability distribution (such as the ones returned by `ordpy.ordinal_distribution()`).
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **probs** (*boolean*) – If *True*, it assumes **data** is an ordinal probability distribution. If *False*, **data** is expected to be a one- or two-dimensional array (default: *False*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns

Values of normalized permutation entropy and Fisher information.

Return type

tuple

Examples

```
>>> fisher_shannon([4,7,9,10,6,11,3], dx=2)
(0.9182958340544896, 0.028595479208968325)
>>>
>>> p = ordinal_distribution([4,7,9,10,6,11,3], dx=2, return_missing=True)[1]
>>> fisher_shannon(p, dx=2, probs=True)
(0.9182958340544896, 0.028595479208968325)
>>>
>>> fisher_shannon([[1,2,1],[8,3,4],[6,7,5]], dx=2, dy=2)
(0.32715643797829735, 1.0)
>>>
>>> fisher_shannon([1/3, 1/15, 2/15, 3/15, 4/15, 0], dx=3, probs=True)
(0.8314454838586239, 0.19574180681374548)
>>>
>>> fisher_shannon([[1,2,1,4],[8,3,4,5],[6,7,5,6]], dx=3, dy=2)
(0.2107070115500801, 1.0)
```

ordpy.**global_node_entropy**(*data*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *overlapping=True*, *connections='all'*, *tie_precision=None*)

Calculates global node entropy[Page 1, 11,Page 1, 13] for an ordinal network obtained from data. (Assumes directed and weighted edges).

Parameters

- **data** (array, return of *ordpy.ordinal_network()*) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ or an ordinal network returned by *ordpy.ordinal_network()**[0].
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **overlapping** (*boolean*) – If *True*, **data** is partitioned into overlapping sliding windows (default: *True*). If *False*, adjacent partitions are non-overlapping.
- **connections** (*str*) – The ordinal network is constructed using *'all'* permutation successions in a symbolic sequence or only *'horizontal'* or *'vertical'* successions. Parameter only valid for image data (default: *'all'*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns
Value of global node entropy.

Return type
float

Notes

Examples

```
>>> global_node_entropy([1,2,3,4,5,6,7,8,9], dx=2)
0.0
>>>
>>> global_node_entropy(ordinal_network([1,2,3,4,5,6,7,8,9], dx=2))
0.0
>>>
>>> global_node_entropy(np.random.uniform(size=100000), dx=3)
1.4988332319747597
>>>
>>> global_node_entropy(random_ordinal_network(dx=3))
1.5
>>>
>>> global_node_entropy([[1,2,1,4],[8,3,4,5],[6,7,5,6]], dx=2, dy=2, connections=
↪'horizontal')
0.25
>>>
>>> global_node_entropy([[1,2,1,4],[8,3,4,5],[6,7,5,6]], dx=2, dy=2, connections=
↪'vertical')
0.0
```

[0] In case **data** is an ordinal network returned by *ordpy.ordinal_network()*, the parameters of *ordpy.global_node_entropy()* are inferred from the network.

`ordpy.`**`logq`**(*x*, *q=1*)

Calculates the q-logarithm of x.

Parameters

- **x** (*float, array*) – Real number or array containing real numbers.
- **q** (*float*) – Tsallis's q parameter (default: 1).

Returns
Value or array of values containing the q-logarithm of x.

Return type
float or array

Notes

The q-logarithm of x is defined as†[0]

log_q (x) = frac{x^{1-q} - 1}{1-q}~~text{for}~~qneq 1

and $\log_q(x) = \log(x)$ for $q = 1$.

Examples

```
>>> logq(np.math.e)
1.0
>>>
>>> logq([np.math.e for i in range(5)])
array([1., 1., 1., 1., 1.])
```

`ordpy.`**`maximum_complexity_entropy`**(*dx=3*, *dy=1*, *m=1*)

Generates data corresponding to values of normalized permutation entropy and statistical complexity which delimit the upper boundary in the complexity-entropy causality plane[20],‡[0].

Parameters

- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); must be 1 for time series (default: 1).
- **m** (*int*) – The length of the returned array containing values of permutation entropy and statistical complexity is given by $[(d_x \times d_y)! - 1] \times m$.

Returns
Values of normalized permutation entropy and statistical complexity belonging to the upper limiting curve of the complexity-entropy causality plane.

Return type
array

[0] Tsallis, C. (2009). Introduction to Nonextensive Statistical Mechanics: Approaching a Complex World. Springer.
[20] Martin, M. T., Plastino, A., & Rosso, O. A. (2006). Generalized Statistical Complexity Measures: Geometrical and Analytical Properties, Physica A, 369, 439–462.
[0] This function was adapted from Sebastian Sippel et al., statcomp: Statistical Complexity and Information Measures for Time Series Analysis, version 0.1.0. (Computer Software in R).

Notes

Examples

```
>>> maximum_complexity_entropy(dx=3, dy=1, m=1)
array([[-0.        , -0.        ],
       [ 0.38685281,  0.27123863],
       [ 0.61314719,  0.29145164],
       [ 0.77370561,  0.22551573],
       [ 0.8982444 ,  0.12181148]])
>>>
>>> maximum_complexity_entropy(dx=3, dy=1, m=2)
array([[-0.        , -0.        ],
       [ 0.251463  ,  0.19519391],
       [ 0.38685281,  0.27123863],
       [ 0.57384034,  0.28903864],
       [ 0.61314719,  0.29145164],
       [ 0.76241899,  0.2294088 ],
       [ 0.77370561,  0.22551573],
       [ 0.89621768,  0.12320718],
       [ 0.8982444 ,  0.12181148],
       [ 1.        ,  0.        ]])
```

ordpy.**minimum_complexity_entropy**(*dx=3*, *dy=1*, *size=100*)

Generates data corresponding to values of normalized permutation entropy and statistical complexity which delimit the lower boundary in the complexity-entropy causality plane[Page 12, 20,159].

Parameters

- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); must be 1 for time series (default: 1).
- **size** (*int*) – The length of the array returned containing pairs of values of permutation entropy and statistical complexity.

Returns

Values of normalized permutation entropy and statistical complexity belonging to the lower limiting curve of the complexity-entropy causality plane.

Return type

array

Notes

Examples

```
>>> minimum_complexity_entropy(dx=3, size=10)
array([[-0.        , -0.        ],
       [ 0.25534537,  0.17487933],
       [ 0.43376898,  0.21798915],
       [ 0.57926767,  0.21217508],
```

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[159] This function was adapted from Sebastian Sippel et al., statcomp: Statistical Complexity and Information Measures for Time Series Analysis, version 0.1.0. (Computer Software in R).

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```
       [ 0.70056313,  0.18118876],
       [ 0.80117413,  0.1378572 ],
       [ 0.88242522,  0.09085847],
       [ 0.94417789,  0.04723328],
       [ 0.98476216,  0.01396083],
       [ 1.        ,  0.        ]])
>>>
>>> minimum_complexity_entropy(dx=4, size=5)
array([[-0.00000000e+00, -0.00000000e+00],
       [ 4.09625322e-01,  2.10690777e-01],
       [ 6.90580872e-01,  1.84011590e-01],
       [ 8.96072453e-01,  8.64190054e-02],
       [ 1.00000000e+00, -3.66606083e-16]])
```

ordpy.**missing_links**(*data*, *dx=3*, *dy=1*, *return_fraction=True*, *return_missing=True*, *tie_precision=None*)

Identifies transitions between ordinal patterns (permutations) not occurring in data. (These transtitions correspond to directed edges in ordinal networks[Page 1, 11].) Assumes overlapping windows and unitary embedding delay. In case dx>1 and dy>1, both 'horizontal' and 'vertical' connections are considered (see `ordpy.ordinal_network()`).

Parameters

- **data** (array, return of `ordpy.ordinal_network()`) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ or the nodes, edges and edge weights returned by `ordpy.ordinal_network()`.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3)
- **dy** (*int*) – Embedding dimension (vertical axis); must be 1 for time series (default: 1).
- **return_fraction** (*boolean*) – if *True*, returns the fraction of missing links among ordinal patterns relative to the total number of possible links (transitions) for given values of **dx** and **dy** (default: *True*). If *False*, returns the raw number of missing links.
- **return_missing** (*boolean*) – if True, returns the missing links in **data**; if False, it only returns the fraction/number of these missing links.
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns

Tuple containing an array and a float indicating missing links and their relative fraction (or raw number).

Return type

tuple

Examples

```
>>> missing_links([4,7,9,10,6,11,3], dx=2, return_fraction=False)
(array([['1|0', '1|0']], dtype='<U3'), 1)
>>>
>>> missing_links(ordinal_network([4,7,9,10,6,11,3], dx=2), dx=2, return_
↪fraction=True)
(array([['1|0', '1|0']], dtype='<U3'), 0.25)
>>>
>>> missing_links([4,7,9,10,6,11,3,5,6,2,3,1], dx=3, return_fraction=False)
(array([['0|1|2', '0|2|1'],
        ['0|2|1', '1|0|2'],
        ['0|2|1', '1|2|0'],
        ['0|2|1', '2|1|0'],
        ['1|0|2', '0|1|2'],
        ['1|0|2', '0|2|1'],
        ['1|2|0', '0|2|1'],
        ['2|0|1', '2|1|0'],
        ['2|1|0', '1|0|2'],
        ['2|1|0', '1|2|0'],
        ['2|1|0', '2|1|0']], dtype='<U5'), 11)
```

ordpy.**missing_patterns**(*data*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *return_fraction=True*, *return_missing=True*, *probs=False*, *tie_precision=None*)

Identifies ordinal patterns (permutations) not occurring in data[Page 1, 14].

Parameters

- **data** (array, return of *ordpy.ordinal_distribution()*) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ or the ordinal patterns and probabilities returned by *ordpy.ordinal_distribution()* with *return_missing=True*.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **return_fraction** (*boolean*) – if *True*, returns the fraction of missing ordinal patterns relative to the total number of ordinal patterns for given values of **dx** and **dy** (default: *True*); if *False*, returns the raw number of missing patterns.
- **return_missing** (*boolean*) – if *True*, returns the missing ordinal patterns in **data** (default: *True*); if *False*, it only returns the fraction/number of these missing patterns.
- **probs** (*boolean*) – If *True*, assumes **data** to be the return of *ordpy.ordinal_distribution()* with *return_missing=True*. If *False*, **data** is expected to be a one- or two-dimensional array (default: *False*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns

Tuple containing an array and a float indicating missing ordinal patterns and their relative fraction (or raw number).

Return type
tuple

Examples

```
>>> missing_patterns([4,7,9,10,6,11,3], dx=2, return_fraction=False)
(array([], shape=(0, 2), dtype=int64), 0)
>>>
>>> missing_patterns([4,7,9,10,6,11,3,5,6,2,3,1], dx=3, return_fraction=True,␣
↪return_missing=False)
0.3333333333333333
>>>
>>> missing_patterns(ordinal_distribution([4,7,9,10,6,11,3], dx=2, return_
↪missing=True), dx=2, probs=True)
(array([], shape=(0, 2), dtype=int64), 0.0)
>>>
>>> missing_patterns(ordinal_distribution([4,7,9,10,6,11,3], dx=3, return_
↪missing=True), dx=3, probs=True)
(array([[0, 2, 1],
        [1, 2, 0],
        [2, 1, 0]]), 0.5)
```

ordpy.**ordinal_distribution**(*data*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *return_missing=False*, *tie_precision=None*, *ordered=False*)

Applies the Bandt and Pompe[Page 1, 2] symbolization approach to obtain a probability distribution of ordinal patterns (permutations) from data.

Parameters

- **data** (*array*) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **return_missing** (*boolean*) – If *True*, it returns ordinal patterns not appearing in the symbolic sequence obtained from **data**. If *False*, these missing patterns (permutations) are omitted (default: *False*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).
- **ordered** (*boolean*) – If *True*, it also returns ordinal patterns not appearing in the symbolic sequence obtained from **data** in ascending ordered. The *return_missing* parameter must also be *True*.

Returns
Tuple containing two arrays, one with the ordinal patterns and another with their corresponding probabilities.

Return type
tuple

Examples

```
>>> ordinal_distribution([4,7,9,10,6,11,3], dx=2)
(array([[0, 1],
        [1, 0]]),
 array([0.66666667, 0.33333333]))
>>>
>>> ordinal_distribution([4,7,9,10,6,11,3], dx=3, return_missing=True)
(array([[0, 1, 2],
        [1, 0, 2],
        [2, 0, 1],
        [0, 2, 1],
        [1, 2, 0],
        [2, 1, 0]]),
 array([0.4, 0.2, 0.4, 0. , 0. , 0. ]))
>>>
>>> ordinal_distribution([[1,2,1],[8,3,4],[6,7,5]], dx=2, dy=2)
(array([[0, 1, 3, 2],
        [1, 0, 2, 3],
        [1, 2, 3, 0]]),
 array([0.5 , 0.25, 0.25]))
>>>
>>> ordinal_distribution([[1,2,1,4],[8,3,4,5],[6,7,5,6]], dx=2, dy=2, taux=2)
(array([[0, 1, 3, 2],
        [0, 2, 1, 3],
        [1, 3, 2, 0]]),
 array([0.5 , 0.25, 0.25]))
```

`ordpy.`**`ordinal_network`**(*data*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *normalized=True*, *overlapping=True*, *directed=True*, *connections='all'*, *tie_precision=None*)

Maps a data set into the elements (nodes, edges and edge weights) of its corresponding ordinal network representation[Page 1, 10,Page 1, 11,Page 1, 12].

Parameters

- **data** (*array*) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **normalized** (*boolean*) – If *True*, edge weights represent transition probabilities between permutations (default: *True*). If *False*, edge weights are transition counts.
- **overlapping** (*boolean*) – If *True*, **data** is partitioned into overlapping sliding windows (default: *True*). If *False*, adjacent partitions are non-overlapping.
- **directed** (*boolean*) – If *True*, ordinal network edges are directed (default: *True*). If *False*, edges are undirected.
- **connections** (*str*) – The ordinal network is constructed using *'all'* permutation successions in a symbolic sequence or only *'horizontal'* or *'vertical'* successions. Parameter only valid for image data (default: *'all'*).

- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns
Tuple containing three arrays corresponding to nodes, edges and edge weights of an ordinal network.

Return type
tuple

Examples

```
>>> ordinal_network([4,7,9,10,6,11,8,3,7], dx=2, normalized=False)
(array(['0|1', '1|0'], dtype='<U3'),
 array([['0|1', '0|1'],
        ['0|1', '1|0'],
        ['1|0', '0|1'],
        ['1|0', '1|0']], dtype='<U3'),
 array([2, 2, 2, 1]))
>>>
>>> ordinal_network([4,7,9,10,6,11,8,3,7], dx=2, overlapping=False,␣
↪normalized=False)
(array(['0|1', '1|0'], dtype='<U3'),
 array([['0|1', '0|1'],
        ['0|1', '1|0']], dtype='<U3'),
 array([2, 1]))
>>>
>>> ordinal_network([[1,2,1],[8,3,4],[6,7,5]], dx=2, dy=2, normalized=False)
(array(['0|1|3|2', '1|0|2|3', '1|2|3|0'], dtype='<U7'),
 array([['0|1|3|2', '1|0|2|3'],
        ['0|1|3|2', '1|2|3|0'],
        ['1|0|2|3', '0|1|3|2'],
        ['1|2|3|0', '0|1|3|2']], dtype='<U7'),
 array([1, 1, 1, 1]))
>>>
>>> ordinal_network([[1,2,1],[8,3,4],[6,7,5]], dx=2, dy=2, normalized=True,␣
↪connections='horizontal')
(array(['0|1|3|2', '1|0|2|3', '1|2|3|0'], dtype='<U7'),
 array([['0|1|3|2', '1|0|2|3'],
        ['1|2|3|0', '0|1|3|2']], dtype='<U7'),
 array([0.5, 0.5]))
```

ordpy.**ordinal_sequence**(*data*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *overlapping=True*, *tie_precision=None*)

Applies the Bandt and Pompe[Page 1, 2] symbolization approach to obtain a sequence of ordinal patterns (permutations) from data.

Parameters

- **data** (*array*) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ $(n \times m)$.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).

- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **overlapping** (*boolean*) – If *True*, **data** is partitioned into overlapping sliding windows (default: *True*). If *False*, adjacent partitions are non-overlapping.
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* number of decimals (default: *None*).

Returns
Array containing the sequence of ordinal patterns.

Return type
array

Examples

```
>>> ordinal_sequence([4,7,9,10,6,11,3], dx=2)
array([[0, 1],
       [0, 1],
       [0, 1],
       [1, 0],
       [0, 1],
       [1, 0]])
>>>
>>> ordinal_sequence([4,7,9,10,6,11,3], dx=2, taux=2)
array([[0, 1],
       [0, 1],
       [1, 0],
       [0, 1],
       [1, 0]])
>>>
>>> ordinal_sequence([[1,2,1,4],[8,3,4,5],[6,7,5,6]], dx=2, dy=2)
array([[[0, 1, 3, 2],
        [1, 0, 2, 3],
        [0, 1, 2, 3]],
       [[1, 2, 3, 0],
        [0, 1, 3, 2],
        [0, 1, 2, 3]]])
>>>
>>> ordinal_sequence([1.3, 1.2], dx=2), ordinal_sequence([1.3, 1.2], dx=2, tie_
↪precision=0)
(array([[1, 0]]), array([[0, 1]]))
```

ordpy.**permutation_contrasts**(*data*, *taux=1*, *tie_precision=None*)

Calculates the four pattern (permutation) contrasts[Page 2, 18] of a time series (with dx=3) using an ordinal distribution obtained from data.

Parameters

- **data** (*array*) – Array object containing a pair of arrays in the format $[x_1, x_2, x_3, \ldots, x_n]$.
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns
Tuple containing values corresponding to the pattern (permutation) contrasts: up-down balance, persistence, rotational-asymmetry, and up-down scaling.

Return type
tuple

Examples

```
>>> permutation_contrasts([4,7,9,10,6,11,3])
(0.4, 0.4, 0.6, -0.2)
>>>
>>> permutation_contrasts([4,7,9,10,6,11,3], taux=2)
(0.0, 0.666, -0.333, 0.333)
>>>
>>> permutation_contrasts([5,2,3,4,2,7,4])
(0.2, 0.2, 0.0, 0.0)
>>>
>>> permutation_contrasts([5, 2, 3, 4, 2, 7, 4], taux=2)
(0.0, 0.666, 0.333, 0.333)
```

`ordpy.`**`permutation_entropy`**(*data*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *base=2*, *normalized=True*, *probs=False*, *tie_precision=None*)

Calculates the Shannon entropy using an ordinal distribution obtained from data[Page 1, 2, Page 1, 3].

Parameters

- **data** (*array*) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ or an ordinal probability distribution (such as the ones returned by `ordpy.ordinal_distribution()`).
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **base** (*str, int*) – Logarithm base in Shannon's entropy. Either 'e' or 2 (default: 2).
- **normalized** (*boolean*) – If *True*, permutation entropy is normalized by its maximum value (default: *True*). If *False*, it is not.
- **probs** (*boolean*) – If *True*, it assumes **data** is an ordinal probability distribution. If *False*, **data** is expected to be a one- or two-dimensional array (default: *False*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns
Value of permutation entropy.

Return type
float

Examples

```
>>> permutation_entropy([4,7,9,10,6,11,3], dx=2, base=2, normalized=True)
0.9182958340544896
>>>
>>> permutation_entropy([.5,.5], dx=2, base=2, normalized=False, probs=True)
1.0
>>>
>>> permutation_entropy([[1,2,1],[8,3,4],[6,7,5]], dx=2, dy=2, base=2,␣
↪normalized=True)
0.32715643797829735
>>>
>>> permutation_entropy([[1,2,1,4],[8,3,4,5],[6,7,5,6]], dx=2, dy=2, taux=2, base='e
↪', normalized=False)
1.0397207708399179
```

ordpy.**permutation_js_distance**(*data*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *base='e'*, *normalized=True*, *tie_precision=None*)

Calculates the permutation Jensen-Shannon distance[Page 2, 17] between multiple time series (or channels of a multivariate time series) or multiple matrices (images) using ordinal distributions obtained from data.

Parameters

- **data** (*array*) – Array object containing arrays in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **base** (*str, int*) – Logarithm base in Shannon's entropy. Either 'e' or 2 (default: 2).
- **normalized** (*boolean*) – If *True*, the permutation Jensen-Shannon distance is normalized by the logarithm of 2. (default: *True*). If *False*, it is not.
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns

Value of permutation Jensen-Shannon distance.

Return type

float

Examples

```
>>> permutation_js_distance([[1,2,6,5,4], [1,2,1]], dx=3, base=2, normalized=False)
0.677604543245723
>>>
>>> permutation_js_distance([[1,2,6,5,4], [1,2,1]], dx=3, base='e',␣
↪normalized=False)
0.5641427870206323
>>>
```

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```
>>> permutation_js_distance([[[1,2,6],[4,2,6]], [[1,2,6],[1,2,4]]], dx=2, dy=2)
0.7071067811865476
>>>
>>> permutation_js_distance([[[1,2,6,5,4],[4,2,6,3,4]], [[1,2,6],[1,2,4]]], dx=2,␣
↪dy=2)
0.809715420521041
```

ordpy.**random_ordinal_network**(*dx=3*, *dy=1*, *overlapping=True*)

Generates the nodes, edges and edge weights of a random ordinal network: the theoretically expected network representation of a random time series[Page 1, 11] or a random two-dimensional array[Page 1, 12]. (Assumes directed edges and unitary embedding delays.)

Parameters

- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **overlapping** (*boolean*) – If *True*, **data** is partitioned into overlapping sliding windows (default: *True*). If *False*, adjacent partitions are non-overlapping.

Returns

Tuple containing three arrays corresponding to nodes, edges and edge weights of the random ordinal network.

Return type

tuple

Examples

```
>>> random_ordinal_network(dx=2)
(array(['0|1', '1|0'], dtype='<U3'),
 array([['0|1', '0|1'],
        ['0|1', '1|0'],
        ['1|0', '0|1'],
        ['1|0', '1|0']], dtype='<U3'),
 array([0.16666667, 0.33333333, 0.33333333, 0.16666667]))
>>>
>>> random_ordinal_network(dx=2, overlapping=False)
(array(['0|1', '1|0'], dtype='<U3'),
 array([['0|1', '0|1'],
        ['0|1', '1|0'],
        ['1|0', '0|1'],
        ['1|0', '1|0']], dtype='<U3'),
 array([0.25, 0.25, 0.25, 0.25]))
```

ordpy.**renyi_complexity_entropy**(*data*, *alpha=1*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *probs=False*, *tie_precision=None*)

Calculates the Rényi normalized permutation entropy and statistical complexity[Page 1, 9] using an ordinal distribution obtained from data.

Parameters

- **data** (*array*) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ or an ordinal probability distribution (such as the ones returned by `ordpy.ordinal_distribution()`).
- **alpha** (*float, array*) – Rényi's *alpha* parameter (default: 1); an array of values is also accepted for this parameter.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **probs** (*boolean*) – If *True*, it assumes **data** is an ordinal probability distribution. If *False*, **data** is expected to be a one- or two-dimensional array (default: *False*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns
Value(s) of normalized permutation entropy and statistical complexity in Rényi's formalism.

Return type
array

Examples

```
>>> renyi_complexity_entropy([4,7,9,10,6,11,3], dx=2)
array([0.91829583, 0.06112817])
>>>
>>> p = ordinal_distribution([4,7,9,10,6,11,3], dx=2, return_missing=True)[1]
>>> renyi_complexity_entropy(p, dx=2, probs=True)
array([0.91829583, 0.06112817])
>>>
>>> renyi_complexity_entropy([4,7,9,10,6,11,3], alpha=2, dx=2)
array([0.84799691, 0.08303895])
>>>
>>> renyi_complexity_entropy([1/3, 1/15, 4/15, 2/15, 1/5, 0], dx=3, probs=True)
array([0.83144548, 0.16576717])
>>>
>>> renyi_complexity_entropy([4,7,9,10,6,11,3], alpha=[1, 2], dx=2)
array([[0.91829583, 0.06112817],
       [0.84799691, 0.08303895]])
>>>
>>> renyi_complexity_entropy([[1,2,1,4],[8,3,4,5],[6,7,5,6]], alpha=3, dx=3, dy=2)
array([0.21070701, 0.20975673])
```

`ordpy.`**`renyi_entropy`**(*data, alpha=1, dx=3, dy=1, taux=1, tauy=1, probs=False, tie_precision=None*)

Calculates the normalized Rényi permutation entropy[Page 1, 9] using an ordinal distribution obtained from data.

Parameters

- **data** (*array*) –

  Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$
  or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ or an ordinal probability distribution (such as the ones returned by

ordpy.ordinal_distribution()).

- **alpha** (*float, array*) – Rényi's *alpha* parameter (default: 1); an array of values is also accepted for this parameter.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **probs** (*boolean*) – If *True*, it assumes **data** is an ordinal probability distribution. If *False*, **data** is expected to be a one- or two-dimensional array (default: *False*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns
Value(s) of the normalized Rényi permutation entropy.

Return type
float, array

Examples

```
>>> renyi_entropy([4,7,9,10,6,11,3], dx=2)
0.9182958340544894
>>>
>>> renyi_entropy([4,7,9,10,6,11,3], alpha=2, dx=2)
0.84799690655495
>>>
>>> renyi_entropy([1/3, 1/15, 4/15, 2/15, 1/5, 0], dx=3, probs=True)
0.8314454838586238
>>>
>>> renyi_entropy([4,7,9,10,6,11,3], alpha=[1,2], dx=2)
array([0.91829583, 0.84799691])
>>>
>>> renyi_entropy([4,7,9,10,6,11,3], alpha=2, dx=3)
0.5701944178769374
```

ordpy.**smoothness_structure**(*data*, *taux=1*, *tauy=1*, *tie_precision=None*)

Calculates the smoothness and curve structure[Page 2, 19] of an image (with embedding parameters dx=dy=2) using an ordinal distribution obtained from data.

Parameters

- **data** (*array*) – Array object in the format $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$.
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns
Values of smoothness and curve structure.

Return type
tuple

Examples

```
>>> smoothness_structure([[1,2,1,4],[8,3,4,5],[6,7,5,6]])
(0.0, 0.333)
>>>
>>> smoothness_structure([[1,2,1,4],[8,3,4,5],[6,7,5,6]], taux=2, tauy=1)
(-0.0833, 0.75)
>>>
>>> smoothness_structure([[1,2,1,4],[8,3,4,5],[6,7,5,6]], taux=2, tauy=2)
(-0.333, 1.0)
```

ordpy.**tsallis_complexity_entropy**(*data*, *q=1*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *probs=False*, *tie_precision=None*)

Calculates the Tsallis normalized permutation entropy and statistical complexity[Page 1, 8] using an ordinal distribution obtained from data.

Parameters

- **data** (*array*) –

  Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$
  or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ or an ordinal probability distribution (such as the ones returned by

  ordpy.ordinal_distribution()).
- **q** (*float, array*) – Tsallis's q parameter (default: 1); an array of values is also accepted for this parameter.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **probs** (*boolean*) – If *True*, it assumes **data** is an ordinal probability distribution. If *False*, **data** is expected to be a one- or two-dimensional array (default: *False*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns
Value(s) of normalized permutation entropy and statistical complexity in Tsallis's formalism.

Return type
array

Examples

```
>>> tsallis_complexity_entropy([4,7,9,10,6,11,3], dx=2)
array([0.91829583, 0.06112817])
>>>
>>> p = ordinal_distribution([4,7,9,10,6,11,3], dx=2, return_missing=True)[1]
>>> tsallis_complexity_entropy(p, dx=2, probs=True)
array([0.91829583, 0.06112817])
>>>
>>> tsallis_complexity_entropy([1/3, 1/15, 4/15, 2/15, 1/5, 0], dx=3, probs=True)
array([0.83144548, 0.16576717])
>>>
>>> tsallis_complexity_entropy([4,7,9,10,6,11,3], dx=2, q=[1,2])
array([[0.91829583, 0.06112817],
       [0.88888889, 0.07619048]])
>>>
>>> tsallis_complexity_entropy([4,7,9,10,6,11,3], q=2, dx=2)
array([0.88888889, 0.07619048])
>>>
>>> tsallis_complexity_entropy([[1,2,1,4],[8,3,4,5],[6,7,5,6]], q=3, dx=3, dy=2)
array([0.93750181, 0.92972165])
```

ordpy.**tsallis_entropy**(*data*, *q=1*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *probs=False*, *tie_precision=None*)

Calculates the normalized Tsallis permutation entropy[Page 1, 8] using an ordinal distribution obtained from data.

Parameters

- **data** (*array*) –

  Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$ or an ordinal probability distribution (such as the ones returned by

  ordpy.ordinal_distribution()).
- **q** (*float or array*) – Tsallis's q parameter (default: 1); an array of values is also accepted for this parameter.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); it must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **probs** (*boolean*) – If *True*, it assumes **data** is an ordinal probability distribution. If *False*, **data** is expected to be a one- or two-dimensional array (default: *False*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns

Value(s) of the normalized Tsallis permutation entropy.

Return type

float, array

Examples

```
>>> tsallis_entropy([4,7,9,10,6,11,3], dx=2)
0.9182958340544894
>>>
>>> tsallis_entropy([4,7,9,10,6,11,3], q=[1,2], dx=2)
array([0.91829583, 0.88888889])
>>>
>>> tsallis_entropy([4,7,9,10,6,11,3], q=2, dx=2)
0.888888888888889
>>>
>>> tsallis_entropy([1/3, 1/15, 4/15, 2/15, 1/5, 0], dx=3, probs=True)
0.8314454838586238
>>>
>>> tsallis_entropy([4,7,9,10,6,11,3], q=2, dx=3)
0.768
```

ordpy.**weighted_permutation_entropy**(*data*, *dx=3*, *dy=1*, *taux=1*, *tauy=1*, *base=2*, *normalized=True*, *tie_precision=None*)

Calculates Shannon entropy using a weighted ordinal distribution obtained from data[Page 2, 15].

Parameters

- **data** (*array*) – Array object in the format $[x_1, x_2, x_3, \ldots, x_n]$ or $[[x_{11}, x_{12}, x_{13}, \ldots, x_{1m}], \ldots, [x_{n1}, x_{n2}, x_{n3}, \ldots, x_{nm}]]$.
- **dx** (*int*) – Embedding dimension (horizontal axis) (default: 3).
- **dy** (*int*) – Embedding dimension (vertical axis); must be 1 for time series (default: 1).
- **taux** (*int*) – Embedding delay (horizontal axis) (default: 1).
- **tauy** (*int*) – Embedding delay (vertical axis) (default: 1).
- **base** (*str, int*) – Logarithm base in Shannon's entropy. Either 'e' or 2 (default: 'e').
- **normalized** (*boolean*) – If *True*, weighted permutation entropy is normalized by its maximum value. If *False*, it does not (default: *True*).
- **tie_precision** (*None, int*) – If not *None*, **data** is rounded with *tie_precision* decimal numbers (default: *None*).

Returns
The value of weighted permutation entropy.

Return type
float

Examples

```
>>> weighted_permutation_entropy([4,7,9,10,6,11,3], dx=2, base=2, normalized=False)
0.9125914261094841
>>>
>>> weighted_permutation_entropy([[1,2,1],[8,3,4],[6,7,5]], dx=2, dy=2, base=2)
0.2613186822347165
>>>
>>> weighted_permutation_entropy([[1,2,1,4],[8,3,4,5],[6,7,5,6]], dx=2, dy=2,␣
```

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```
↪taux=2, base='e')
0.22725901135766047
```

CHAPTER FOUR

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